Higher Spin Field Theory

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Motivations

- Higher Spin (HS) Theory has a long history ~ 80 years
  - had to overcome important conceptual issues (interaction problem)
  - study of these fundamental problems may have at least academic interest
- since the early 50's HS particles have been observed in Nature as hadron resonances with s ≤ 11/2

$$M_s^2 \sim \frac{1}{\alpha'}(s-a) \qquad \Rightarrow \qquad \text{String Theory}$$

- HS fields constitute a major (infinite-dimensional) subsector of String Theory
  - they are in charge of a soft UV behavior or even finiteness of String Theory
- AdS/CFT correspondence
  - **tensionless string** v.s. 4D free SYM *(...Sundborg, Witten, Sezgin & Sundell, Bianchi et. al,...)*
  - **non- susy** duality between AdS_4 massless HS and 3d CFT models (*Sezgin & Sundell, Klebanov & Polyakov* 2002, ...,*Petkou* 2003,..., Giombi & Yin 2009-2011)
  - HS gravity AdS_3/CFT_2 duality (..., *Gaberdiel & Gopakumar 2010,...*)

Plan of the talk

- Short history
- Overview of the main problem – consistent HS interactions
- Way to solve the interaction problem
  - group-theoretical and geometrical foundation of HS theory
  - Vasiliev construction of non-linear Higher-Spin systems
- Higher spins, strings and AdS/CFT

Historical remarks

- First appearance of higher-spin fields
  - in 1932 Majorana proposed a field equation which describes infinite tower of HS fields with inverse mass-spin dependence

$$M_s \sim \frac{1}{s+\frac{1}{2}}$$

  - in 1936 Dirac considered (non-Lagrangian) equations for a single free field of an arbitrary (half)-integer spin

$$(\partial^2 - M^2)\varphi_{m_1\cdots m_s} = 0, \qquad \partial^{m_1}\varphi_{m_1\cdots m_s} = 0, \qquad \eta^{m_1 m_2}\varphi_{m_1 m_2\cdots m_s} = 0 \qquad (m = 0,1,\ldots,D-1)$$

$$\text{e.g. spin-1 field} \quad (\partial^2 - M^2)A_m = 0, \qquad \partial^m A_m = 0 \qquad\qquad D = 4$$

$$Fermions: \quad (i\gamma^m\partial_m - M)\psi_{m_1\cdots m_{s-\frac{1}{2}}} = 0, \qquad \partial^{m_1}\psi_{m_1\cdots m_{s-\frac{1}{2}}} = 0, \qquad \gamma^{m_1}\psi_{m_1\cdots m_{s-\frac{1}{2}}} = 0$$

  - in 1939 Fierz and Pauli addressed the problem of coupling HS fields with s>1 to an electro-magnetic field – **first serious problem of HS theory**

Historical remarks

- 1939 Fierz-Pauli construction
  - problem with the introduction of the minimal EM coupling into the Dirac eqs.

$$\partial_m \;\Rightarrow\; \partial_m - A_m$$

  - Reason: non-Lagrangian form of the higher-spin Dirac eqs.
  - Cure: lift the system to a Lagrangian and introduce EM coupling therein
  - a # of auxiliary fields of lower spins is required
  - Fierz & Pauli considered in detail only the cases of s=2 and 3/2, massive and massless. For arbitrary s they gave only a general prescription of how to construct a Lagrangian realized by Singh & Hagen in 1974 for massive fields. Lagrangians for m=0 HS fields were constructed by Fronsdal & Fang in 1978
- F&P spin 3/2 Lagrangian description requires an auxiliary spinor field ρ^α

$$\psi_m^\alpha \;\; (\gamma^m \psi_m = 0) \;+\; \rho^\alpha \;\;\Rightarrow\;\; \Psi_m^\alpha = \psi_m^\alpha + (\gamma_m \rho)^\alpha \;\; \text{- Rarita-Schwinger field (1941)}$$

- Fierz & Pauli noticed fermionic gauge symmetry of the massless 3/2 field

$$\delta\psi_m^\alpha = \partial_m \xi^\alpha(x) - \tfrac{1}{4}(\gamma_m \gamma^n \partial_n \xi)^\alpha, \quad \delta\rho^\alpha = \tfrac{1}{4}(\gamma^n \partial_n \xi)^\alpha \quad \Rightarrow \quad \delta\Psi_m^\alpha = \partial_m \xi^\alpha$$

- In 1939 Fierz & Pauli were about to discover supergravity (and susy) if they decided to couple spin-3/2 field to gravity

Fierz & Pauli, Proc. R. Soc. Lon. 1939, p. 226

Whereas the theory for the spin value 2 has an important generalization for force fields, namely the gravitational theory, we here have no such connexion with a known theory. To get a generalization of the theory with interactions, one would first of all have to find a physical interpretation of the gauge group, and of the conservation theorem connected with this group.

It took more than 30 years to gain understanding of the spin-3/2 gauge symmetry which resulted in the construction of Supergravity

- *D. Volkov & V. Soroka,* Higgs Effect for Goldstone Particles with Spin 1/2, JETP Letters, **18**, 312, 1973 (supersymmetry is non-linearly realized)
- *D. Z. Freedman, P. van Nieuwenhuizen & S. Ferrara*, Progress Toward a Theory of Supergravity, Phys. Rev. **D 13**, 3214, 1976
- *S. Deser & B. Zumino*, Consistent Supergravity, Phys. Lett. **B 62**, 335, 1976

Problems of consistent interactions of HS fields

- No-go theorems regarding *S*-matrix amplitudes of interacting massless HS particles in $D>3$
  - 1964, Weinberg – non gauge invariant behavior of amplitudes describing the emission of soft massless HS particles (no long-range HS interactions)
  - 1967, Coleman-Mandula theorem – the *S*-matrix does not allow for HS symmetries
  - 1980, Weinber & Witten – matrix elements of the energy-momentum between two massless HS states should vanish
  - Later refinements of these and other theorems: Haag, Lopuszanski & Sohnius '75, …, Benincasa & Cachazo '07, Porrati '08,…
- Claim incompatibility of interactions of massless HS fields with general consistency requirements of the relativistic scattering theory
- 1979, Aragone & Deser – minimal Lagrangian coupling of massless HS fields to gravity is incompatible with HS gauge symmetry

Limitations of the no-go theorems

- Assumption of flat (Minkowski space) vacuum
- Conventional minimal (e.g. gravity) interactions with not more than 2 derivatives
- Finite number of fields involved

If any or all of these assumptions are relaxed, the no-go theorems can be circumvented (for a review *see Bekaert, Boulanger & Sundell,* arXiv 1007.0435)

$$\varphi\partial^{s}\varphi\partial^{s'}\varphi$$

Examples:

-Cubic HS vertices with higher (> s) derivatives do exist in Minkowski and (A)dS spaces

(*Bengtsson, Bengtsson & Brink* '83, *Berends, Burgers & van Dam* '85,..., *Fradkin & Vasiliev '87*, ..., *Metsaev* '05, ...)

-String theory on a flat background gives rise to a consistent theory of an *infinite* number of interacting **massive** HS fields $M_s^2 \sim T(s-1)$

- HS vertices from string theory (e.g. *Sagnotti & Taronna* '10, *Taronna* '11)

Questions: Whether simpler interacting HS systems exist, e.g. with M=0?

Whether String Theory can be considered as a spontaneously broken phase of a more fundamental HS gauge theory?

$$T \sim \tfrac{1}{\alpha'} \;\rightarrow\; 0$$

Gauge Theory of massless HS fields

- Fradkin & Vasiliev 1987: coupling of massless HS fields to gravity and among themselves is possible in backgrounds with non-zero cosmological constant (*e.g. in* (*A*)*dS*)

consistency requires: higher-derivative terms and an infinite # of fields
- flat limit of these theories is singular

$$L_{\text{int}} = \frac{\varphi \partial^k \varphi \varphi}{\Lambda^{k/2}} \to \infty \quad \text{at} \quad \Lambda \to 0;$$ Λ plays the role similar to a mass

- simplest case: symmetric HS fields $\varphi_{m_1 \cdots m_s}, \quad \psi^{\alpha}_{m_1 \cdots m_{s-\frac{1}{2}}}$
  (first Regge trajectory in String Theory)

- mixed-symmetry HS fields in D>4 $\varphi_{m_1 \cdots m_i, n_1 \cdots n_k, \cdots}, \quad \psi^{\alpha}_{m_1 \cdots m_i, n_1 \cdots n_k, \cdots}$
  (not to be discussed)

Gauge Theory of massless HS fields

- Lagrangian description of free massless symmetric HS fields
  1978, Fronsdal (bosons), Fang & Fronsdal (fermions)
  - Abelian gauge symmetry: $\delta\varphi_{m_1\cdots m_s} = \partial_{(m_1}\xi_{m_2\cdots m_s)}, \quad \delta\psi^{\alpha}_{m_1\cdots m_{s-\frac{1}{2}}} = \partial_{(m_1}\xi^{\alpha}_{m_2\cdots m_{s-\frac{1}{2}})}$
  - traceless conditions:

$$\varphi^{mnpq}{}_{mnpqm_5\cdots m_s} = 0, \quad \gamma^m\gamma^p\gamma^q\psi_{mpqm_4\cdots m_{s-\frac{1}{2}}} = 0, \quad \mathrm{tr}\xi = 0, \quad \gamma^m\xi_{mm_2\cdots m_{s-\frac{1}{2}}} = 0$$

Field equations: $$G_{m_1\cdots m_s} \equiv \partial^2\varphi_{m_1\cdots m_s} - \partial_{(m_1}\partial^n\varphi_{m_2\cdots m_s)n} + \partial_{(m_1m_2}\varphi_{m_3\cdots m_s)n}{}^n = 0$$

spin-1: $$\partial^2 A_m - \partial_m\partial^n A_n = 0$$

Lagrangian: $$L = \tfrac{1}{2}\varphi^{m_1\cdots m_s}G_{m_1\cdots m_s} - \tfrac{1}{8}s(s-1)\varphi_p{}^{pm_3\cdots m_s}G_{m_3\cdots m_s n}{}^n$$

Coupling of massless HS fields to gravity

spin-5/2 example

Free Lagrangian: $$L_{5/2} = -\tfrac{1}{2}\psi^{mn}G_{mn} + \tfrac{5}{8}\psi^{mn}\gamma_n\gamma^p G_{mp} + \tfrac{15}{32}\psi^m{}_m G^n{}_n$$

Equations of motion: $$G_{mn} \equiv i\gamma^p(\partial_p\psi_{mn} - \partial_m\psi_{pn} - \partial_n\psi_{mp}) = 0$$

Symmetry: $$\delta\psi^\alpha_{mn} = \partial_m\xi^\alpha_n + \partial_n\xi^\alpha_m, \quad \gamma^m\xi_m = 0 \;\Rightarrow\; \delta L_{5/2} = 0 = \delta G_{mn}$$

Minimal coupling to gravity: $$\partial_m \;\Rightarrow\; D_m = \partial_m + \Gamma^p_{mn} + \omega^{\alpha\beta}_m$$

$$G_{mn} \equiv i\gamma^p(D_p\psi_{mn} - D_m\psi_{pn} - D_n\psi_{mp}) = 0, \qquad \delta\psi_{mn} = D_m\xi_n + D_n\xi_m$$

Variation of e.o.m: $$\delta G_{mn} \sim \gamma^p\xi_{(m}R_{n)p} - \underline{R_{p(mn)}}{}^q\gamma^p\xi_q \neq 0$$

Difference from supergravity: $$\delta(\gamma_{mnp}D^n\psi^p) \sim R_{mp}\gamma^p\xi$$ no Riemann tensor (-> susy)

<u>AdS Background</u>: $$G_{mn} \equiv i\gamma^p(\nabla_p\psi_{mn} - \nabla_m\psi_{pn} - \nabla_n\psi_{mp}) - 2\Lambda^{\frac{1}{2}}\psi_{mn} = 0, \quad \nabla_m \text{ - AdS derivative}$$

Higher-derivative non-minimal coupling cancels Riemann tensor in the spin 5/2 eom variation: $$L_{\rm int} = \frac{i\sqrt{g}}{\Lambda}\psi^{mn}R_{pmnq}\nabla\psi^{pq} + \ldots$$

Other formulations of free HS field theory

- Unconstrained formulations
  - with the use of auxiliary fields (*Buchbinder, Pashnev, Tsulaia, Bekaert, Sagnotti, Francia, Galajinsky,…)*
  - non-local formulation with the use of HS curvatures (*Francia & Sagnotti*)

- BRST construction a la String Field Theory (…Pashnev, Tsulaia, Burdik, Buchbinder, Bekaert, Krykhtin, Reshetnyak, Grigoriev, Barnich…)

- "Kaluza-Klein-like" formulation in tensorial spaces (*Fronsdal, Bandos, Lukierski, D.S., Vasiliev, Gelfond, Plyushchay, Tsulaia, Pasti , Tonin, de Azcarraga, Bekaert, Ivanov*…)

- Frame-like formulation (*Vasiliev '80,* …)

turned out to be the most suitable for the development of interacting HS theory

Frame-like formulation and geometry of HS fields

- "Frame-like" indicates that this formulation is analogous to the Cartan-Weyl formulation of gravity in terms of the frame field, or vielbein, and spin connection (naturally based on the diff. form formalism)

$$e^a = dx^m e_m^a(x) \quad \text{- vielbein,} \qquad \omega^{ab} = -\omega^{ab} = dx^m \omega_m^{ab}(x) \quad \text{- spin connection}$$

- Fronsdal formulation is "metric-like". It generalizes the Einstein formulation of General Relativity

$$g_{mn}(x) \quad \Rightarrow \quad \varphi_{m_1 \cdots m_s}(x)$$

- Understanding the group-theoretical structure and geometry underlying a theory is crucial for its development (*e.g.* GR and YM)

For HS fields this should give us a clue to the structure of their interactions

HS geometry in the metric-like formulation

$A_m(x), \quad g_{mn}(x), \ldots, \quad \varphi_{m_1 \cdots m_s}(x)$ - metric-like field potentials

$\delta\varphi_{m_1 \cdots m_s} = \partial_{(m_1} \xi_{m_2 \cdots m_s)}(x)$ - Abelian guage transformations

Christoffel symbols and covariant field strengths (or curvatures)

spin-1: $F_{mn} = \partial_m A_n - \partial_n A_m$

spin-2: $\Gamma_{mn,p} = \frac{1}{2}(\partial_p g_{mn} - \partial_m g_{np} - \partial_n g_{mp}), \qquad \delta\Gamma_{mn,p} = -\partial_m \partial_n \xi_p$

Riemann tensor: (linearized) $R_{mn,pq} = \partial_q \Gamma_{mn,p} - \partial_m \Gamma_{qn,p} + \cdots = \frac{1}{2} \partial_q \partial_p g_{mn} + \cdots, \qquad \delta R_{mn,pq} = 0$

spin-s:

$$\Gamma_{m_1 \ldots m_s, p} = \partial_p \varphi_{m_1 \ldots m_s} - \partial_{m_1} \varphi_{p \ldots m_s} + \cdots$$

$$\Gamma_{m_1 \ldots m_s, p_1 p_2} = \partial_{p_1} \Gamma_{m_1 \ldots m_s, p_1} + \cdots = \partial_{p_1} \partial_{p_2} \varphi_{m_1 \ldots m_s} + \cdots$$

$$\ldots$$

$$R_{m_1 \ldots m_s, p_1 \ldots p_s} = \partial_{p_1} \Gamma_{m_1 \ldots m_s, p_1 \ldots p_{s-1}} + \cdots = \partial_{p_1} \ldots \partial_{p_s} \varphi_{m_1 \ldots m_s} + \cdots, \qquad \delta R_{m_1 \ldots m_s, p_1 \ldots p_s} = 0$$

Generalized Christoffels
(*de Wit & Freedman, 1980*)

HS curvatures appeared alredy in papers by *Bargman & Wigner* (1948) and *Weinberg* (1965)

Fronsdal equation: $G_{m_1 \ldots m_s} = \eta^{p_1 p_2} \Gamma_{m_1 \ldots m_s, p_1 p_2} = \partial^2 \varphi_{m_1 \ldots m_s} + \cdots = 0$

HS geometry in the frame-like formulation

at linearized level

spin-2: $g_{mn} = e^a_m e^b_n \eta_{ab}$, $g_{mn} = \eta_{mn} + \varphi_{mn} = \eta_{mn} + (e_{mn} + e_{nm})$, $\omega^{ab} = dx^m \omega^{ab}_m = -\omega^{ba}$

gauge trans.: $\delta e^a = \delta(dx^m e^a_m) = d\xi^a(x) + dx^b \xi_b{}^a(x)$, $\delta\omega^{ab} = d\xi^{[ab]}(x)$

local Lorentz rotation

gauge invariant quantities: $T^a = de^a + \omega^a{}_b dx^b$, $R^{ab} = d\omega^{ab}$

$T^a = 0$ relates ω^{ab} to e^a

spin-s:

HS vielbein: $\varphi_{m_1 \ldots m_s} \rightarrow dx^m e_m^{a_1 \ldots a_{s-1}}$, $\varphi_{m_1 \ldots m_s} = e_{(m_1 \ldots m_s)}$, $\delta e^{a_1 \ldots a_{s-1}} = d\xi^{a_1 \ldots a_{s-1}} + \xi^{a_1 \ldots a_{s-1}, b} dx_b$

$\operatorname{tr} e^{a_1 a_2 \ldots a_{s-1}} = \eta_{a_1 a_2} e^{a_1 a_2 \ldots a_{s-1}} = 0$ $\qquad \xi^{(a_1 \ldots a_{s-1}, b)} = 0$

HS connections: $\omega^{a_1 \ldots a_{s-1}, b} = dx^m \omega_m^{a_1 \ldots a_{s-1}, b}$, $\omega^{(a_1 \ldots a_{s-1}, b)} = 0$, $\delta\omega^{a_1 \ldots a_{s-1}, b} = d\xi^{a_1 \ldots a_{s-1}, b} + \xi^{a_1 \ldots a_{s-1}, bc} dx_c$

$\omega^{a_1 \ldots a_{s-1}, b_1 b_2}$, $\omega^{(a_1 \ldots a_{s-1}, b_1 b_2)} = 0$, $\delta\omega^{a_1 \ldots a_{s-1}, b_1 b_2} = d\xi^{a_1 \ldots a_{s-1}, b_1 b_2} + \xi^{a_1 \ldots a_{s-1}, b_1 b_2 c} dx_c$

$\ldots$

$\omega^{a_1 \ldots a_{s-1}, b_1 \ldots b_{s-1}}$, $\omega^{(a_1 \ldots a_{s-1}, b_1) b_2 \ldots b_{s-1}} = 0$, $\delta\omega^{a_1 \ldots a_{s-1}, b_1 \ldots b_{s-1}} = d\xi^{a_1 \ldots a_{s-1}, b_1 \ldots b_{s-1}}$

HS geometry in the frame-like formulation

Higher-spin curvatures:

$$R^{a_1\dots a_{s-1}} = de^{a_1\dots a_{s-1}} + \omega^{a_1\dots a_{s-1},b}\, dx_b \quad = 0$$

analogs of $T^a = de^a + \omega^a{}_b dx^b$

$$R^{a_1\dots a_{s-1},b} = d\omega^{a_1\dots a_{s-1},b} + \omega^{a_1\dots a_{s-1},bc}\, dx_c \quad = 0 \quad \longrightarrow$$

contains Fronsdal eqs.

$$\dots$$

$$R^{a_1\dots a_{s-1},b_1\dots b_t} = d\omega^{a_1\dots a_{s-1},b_1\dots b_t} + \omega^{a_1\dots a_{s-1},b_1\dots b_t c}\, dx_c \quad = 0 \quad (t < s-1)$$

$$\dots$$

$$R^{a_1\dots a_{s-1},b_1\dots b_{s-1}} = d\omega^{a_1\dots a_{s-1},b_1\dots b_{s-1}} = dx_{a_s} dx_{b_s} C^{a_1\dots a_s,b_1\dots b_s}(x) \quad \neq 0$$

HS Weyl tensor (tr *C*=0)

Notice the proliferation of indices and quantities required for the description of higher-s fields

Higher spin fields in AdS

AdS geometry:

$$e_0^a, \quad \omega_0^{ab}, \quad R_{AdS}^{ab} = d\omega_0^{ab} + \omega_0^{ac}\omega_{0c}{}^b = \Lambda e_0^a \wedge e_0^b, \quad T_{AdS}^a = de_0^a + e_0^c \omega_{0c}{}^a = 0$$

SO(*2,D-1*) symmetry acts linearly in D+1 dimensions: $V^A V_A = \frac{1}{\Lambda}, \quad A = (0', a), \quad a = 0,1,\dots, D-1$

define flat *SO*(*2,D-1*) connection: $\Omega_0^{AB} = (\sqrt{-\Lambda} e_0^a, \omega_0^{ab}), \quad \Omega_0^{a0'} \equiv \sqrt{-\Lambda} e_0^a, \qquad R_0^{AB} = d\Omega_0^{AB} + \Omega_0^{AC}\Omega_{0C}{}^B = 0$

Free higher spin fields in AdS

AdS HS connection:

$$\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} = (e^{a_1\ldots a_{s-1}},\ldots,\omega^{a_1\ldots a_{s-1},b_1\ldots b_t},\ldots) \qquad (0 < t < s-1)$$

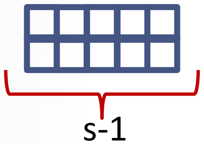
$$e^{a_1\ldots a_{s-1}} = \Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} V_{B_1}\ldots V_{B_{s-1}} = \frac{1}{(-\Lambda)^{\frac{s-1}{2}}}\Omega^{A_1\ldots A_{s-1},0'\ldots 0'_{s-1}}$$

compensator

$$V_A = \frac{1}{\sqrt{-\Lambda}}\delta_A^{0'}$$

Connection properties:

$O(2,D$-$1)$ Young tableau



$$\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} = \Omega^{(A_1\ldots A_{s-1}),B_1\ldots B_{s-1}} = \Omega^{A_1\ldots A_{s-1},(B_1\ldots B_{s-1})} = (-)^{s-1}\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}$$

$$\Omega^{(A_1\ldots A_{s-1},B_1)\ldots B_{s-1}} = 0, \qquad \mathrm{tr}\,\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} = 0$$

Linearized HS curvature:

$$R^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} = D_0\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}, \qquad D_0 = d + \Omega_0^{AB}, \qquad D_0 D_0 = R_{AdS} - \Lambda e_0 e_0 = 0$$

Abelian gauge invariance:

$$\delta\,\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} = D_0\xi^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}(x)$$

HS equations of motion:

$$\boxed{R^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} V_{B_{s-1}} = 0 \quad \Rightarrow \quad R_{a_1\ldots a_{s-1},b_1\ldots b_{s-1}} = e_0^{a_s} e_0^{b_s} C_{a_1\ldots a_s,b_1\ldots b_s}(x)}$$

To summarize, in AdS spin-s field is described by: $\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}(x), \quad C_{A_1\ldots A_s,B_1\ldots B_s}(x), \quad V_A$

Towards the construction of interacting HS systems

Gauge transformations and algebra

$$\delta\,\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} = D_0\xi^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}(x) \quad \leftrightarrow \quad T^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}$$

Abelian generators

spin-2: $$\delta\,\Omega^{A,B} = D_0\xi^{A,B}(x) \quad \leftrightarrow \quad T^{A,B} = -T^{B,A}$$

non-Abelianization leads to *SO*(2,D-1): $[T^{AB},T^{CD}]=0 \quad \rightarrow \quad [T^{AB},T^{CD}]=\eta^{AC}T^{BD}+\cdots$

For s>2 non-Abelianization results in an **infinite**-dimensional HS algebra:

$$[T_{s_1},T_{s_2}] = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \cdots + T_{s_1-s_2+2} \quad \supset \quad SO(2,D-1)$$

The simplest HS algebra contains only even spins *s=2n*. Corresonding minimal HS theory in AdS_4 is (conjectured to be) dual to the 3*d* *O(N)* vector model

Towards the construction of interacting HS systems

Going out into 'hyperspace' to accomodate an infinite number of gauge parameters and fields

$$x^m \;\to\; (x^m, Y_i^A) \quad i=1,2 \quad A=(0',a) \quad (m=0,1,\ldots,D-1)$$

Pack HS fields into 'hyperfields'

$$\Omega(x,Y)=dx^m\Omega_m(x,Y)=A(x)+\Omega^{A,B}(x)\;Y_A^1Y_B^2+\cdots+\Omega^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}\;Y_{A_1}^1\ldots Y_{A_{s-1}}^1Y_{B_1}^2\ldots Y_{B_{s-1}}^2+\cdots$$

one-form connections

$$\xi(x,Y)=\xi_{U(1)}(x)+\xi^{A,B}(x)\;Y_A^1Y_B^2+\cdots+\xi^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}\;Y_{A_1}^1\ldots Y_{A_{s-1}}^1Y_{B_1}^2\ldots Y_{B_{s-1}}^2+\cdots$$

gauge parameters

$$C(x,Y)=\varphi(x)+F_{A,B}Y_1^AY_2^B+C_{A_1A_2,B_1B_2}Y_1^{A_1}Y_1^{A_2}Y_2^{B_1}Y_2^{B_2}+\cdots+C_{A_1\ldots A_s,B_1\ldots B_s}Y_1^{A_1}\ldots Y_1^{A_s}Y_2^{B_1}\ldots Y_2^{B_s}+\cdots$$

zero-forms (Weyl tensors)

Introduce non-commutativity via the (Weyl-Moyal) $*$-product

$$f*g=f(Y)\,e^{\frac{1}{2}\overleftarrow{\partial}_{Ai}\overrightarrow{\partial}_{Bj}\eta^{AB}\varepsilon^{ij}}\,g(Y), \qquad \partial_A^i=\frac{\partial}{Y_i^A},$$

$$[Y_i^A,Y_j^B]_*\equiv Y_i^A*Y_j^B-Y_j^B*Y_i^A=\eta^{AB}\varepsilon_{ij} \quad\to\quad Y_1^{A_1}\cdots Y_1^{A_{s-1}}Y_2^{B_1}\cdots Y_2^{B_{s-1}}=T^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}}$$

oscillator operators

Non-linear HS field equations

(*Vasiliev* '88-'03, Sezgin & Sundell, ….)

Gauge transformations and curvature:

$$\delta\Omega(x,Y) = d\xi(x,Y) + [\Omega,\xi]_*, \qquad R(x,Y) = d\Omega + \Omega * \Omega, \qquad \delta R(x,Y) = [R,\xi]_*$$ adjoint rep.

$$\delta C(x,Y) = \xi * C - C * \tilde{\xi}, \qquad \tilde{\xi} = \xi(-Y^A V_A)$$ - twisted adjoint representation

One more step – doubling of the oscillator coordiantes: $(x^m, Y_i^A) \to (x^m, Y_i^A, Z_i^A)$

new $*$-product $[Y_i^A, Y_j^B]_* = \eta^{AB}\varepsilon_{ij}, \quad [Z_i^A, Z_j^B]_* = -\eta^{AB}\varepsilon_{ij}, \quad [Y_i^A, Z_j^B]_* = 0$

Enlarged (auxiliary) field content

$$B(x,Y,Z) = C(x,Y) + \cdots$$
$$W(x,Y,Z) = \Omega(x,Y) + \cdots, \quad S(x,Y,Z) = dZ_i^A S_A^i(x,Y,Z) = dZ_i^A Z_A^i + \mathcal{S}(x,Y,Z)$$

one-form connection in Z-directions

$$\mathcal{D} = dx^m \partial_m + W + S, \qquad \mathcal{R} = \mathcal{D} * \mathcal{D}$$

Vasiliev eqs.

$$\tilde{\mathcal{D}} * B = 0, \qquad \mathcal{R} = -\tfrac{1}{2} dZ_A^i dZ_i^A + \frac{1}{\Lambda} dZ_A^i dZ_{iB} V^A V^B \left(B * e^{-2Z_i^A V_A Y^{iA} V_A}\right)$$

HS fields and AdS/CFT correspondence

- String AdS/CFT correspondence is a weak-strong coupling duality

$$\lambda = g_{YM}^2 N = R_{AdS}^4 T_{str}^2, \qquad g_{str} \sim \frac{\lambda}{N}$$

- Classical (super)gravity approximation is valid when $g_{str} \to 0, N \to \infty, T \to \infty, \lambda \to \infty$

  weakly coupled bulk theory – strongly coupled CFT

- Most calculations on the bulk side are in this limit

- If we are interested in the correspondence with a weakly coupled CFT $\lambda \to 0$

  - we should deal with a string on a highly curved AdS $R \to 0$
  - or consider a tensionless string limit $T \to 0$ in which HS string states become massless

  (…,Sundborg, Witten, Sezgin & Sundell ’00-’02,…)

- In free 4d SYM theory there is a closed subsector of states corresponding to twist two operators (Δ-s=2). It should correspond to a massless HS field system in AdS5 which form a closed subsector of the full tensionless string (somewhat analogous to supergravity limit*).* *(Sezgin & Sundell ’01*).

  The full free SYM theory also exhibits a Vasiliev-type HS symmetry (*Sezgin & Sundell ‘02, Bianchi et al* ’03-’05)

- Is there simpler free CFT at large N whose dual is a massless HS theory on AdS?

  (*Sezgin & Sundell, Klebanov & Polyakov* 2002)

Free 3d O(N) vector model vs HS theory on AdS_4

Free O(N) model: $S_0 = \frac{1}{2}\int d^3x \partial_m \Phi^I \partial^m \Phi^I \quad (I = 1,2,\ldots,N)$

Infinite # of conserved O(N) singlet currents

$$J_{m_1 \ldots m_s} = \Phi^I \partial_{m_1} \cdots \partial_{m_s} \Phi^I - \partial_{(m_1} \Phi^I \partial_{m_2} \cdots \partial_{m_s)} \Phi^I + \cdots, \qquad \partial^{m_1} J_{m_1 \ldots m_s} = 0 \leftarrow \partial^m \partial_m \Phi^I = 0$$

$$s = 2,4,\ldots,2n$$

J_s can couple to external fields of spin s=2n:

$$L_J = \Phi^I \Phi^I \varphi_0 + J_{m_1 \ldots m_s} \varphi^{m_1 \ldots m_s}, \quad \delta L_J = 0 \text{ under } \delta\varphi^{m_1 \ldots m_s} = \partial^{(m_1} \xi^{m_2 \ldots m_s)}$$ HS gauge symmetry

Non-Susy HS AdS/CFT correspondence: $Z(\varphi^s) = < e^{\frac{i}{\hbar}\int d^2x(\frac{1}{2}\partial\Phi\partial\Phi + J_s\varphi^s)} > \sim e^{S_{HS}(\varphi)} |_{\partial AdS}$

Evidence: three point functions in the CFT match Vasiliev's cubic couplings

(*...Petkou, Sezgin & Sundell,..., Giombi & Yin*)

Evaluate Vasiliev eqs. to 2nd order: $\nabla^2 \varphi_s = \varphi_{s'} \varphi_{s''} + \ldots$

Other HS AdS/CFT instances

- Interacting 3*d* *O*(*N*) vector model at IR critical point and large *N* *(Klebanov & Polyakov 2002)*

$$S_0 = \int d^3x \left(\frac{1}{2} \partial_m \Phi^I \partial^m \Phi^I + \frac{\lambda}{2N} (\Phi^I \Phi^I)^2 \right)$$

dual to the minimal (s=0,2,4,…) HS theory in AdS_4
with a different boundary condition on the bulk scalar field $\varphi_0(x)$

HS gauge symmetries are broken by the boundary conditions: $\partial J_s = O(\frac{1}{N})$

supports interesting issue: in the bulk the HS symmetry may be broken through loop effects and the HS fields acquire mass
(*Girardello, Porati & Zaffaroni* '02, *Manvelyan, Mkrtchyan & Ruhl* '08, *Giombi & Yin* '11)

- Higher-spin AdS_3 / CFT_2 correspondence – has specific features *(Gaberdiel & Gopakumar, Henneaux & Rey, Campoleoni et.al. '10, …)*
  - massless 3d HS theories do not have propagating local d.o.f. (*Blencowe+ Bergshoeff & Stelle*)
  - involve finite # of HS fields described by *SL*(N;**R**) x *SL*(N;**R**) CS field theories $s \leq N$
  - 2d CFT duals are **interacting** *SU*(*N*) coset WZW models with W_N symmetry in large N limit

Other aspects of HS (A)dS/CFT

- Higher spin realization of the dS/CFT correspondence

  (*Anninos, Hartman & Strominger* '11)

- General aspects of HS holography
  - *Koch, Jevicki, Jin & Rodrigues,* '10-11
  - *Douglas, Mazzucato & Razamat* '10
  - Maldacena & Zhiboedov '11

- . . . . .

Other trends and questions in HS theory

- Looking for exact solutions of non-linear HS equations
  (*Vasiliev et. al., Sezgin & Sundell+Iazeolla,…*)
  - Black holes in 4d HS theory *(Didenko, Matveev & Vasiliev '08-09)*
  - New black holes in 3*d SL*(*N*;R) x *SL*(*N*;R) higher spin theories carrying HS charges (*Gutperle & Kraus + Ammon & Perlmutter 2011*)
- Looking for a complete action principal which would produce Vasiliev eqs.
  - 1987 Fradkin-Vasiliev action contains only cubic couplings and no scalars
  - Recent proposal by Boulanger & Sundell 2010
- Understanding the quantum theory of HS fields
  - knowledge of the action and progress in AdS/CFT may be of great help
- HS gauge symmetry breaking and mass generation
  - Whether string theory is a broken phase of an underlying gauge HS theory?
- Mixing of space-time and HS symmetries may claim a revision of our view on space-time in terms of Riemann geometry (generalizing that of string theory)
  - can this give us more hints of what Quantum Gravity is?

Some higher spin reviews

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- A. Campoleoni, "Metric-like Lagrangian Formulations for Higher-Spin Fields of Mixed Symmetry,'' Riv. Nuovo Cim **033** (2010) 123 [arXiv:0910.3155 [hep-th]]
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Спасибо!
Thank you!
Grazie!
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